

# Entangling photons using a charged quantum dot in a microcavity

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We present two novel schemes to generate photon polarization entanglement via single electron spins confined in charged quantum dots inside microcavities. One scheme is via entangled remote electron spins followed by negatively-charged exciton emissions, and another scheme is via a single electron spin followed by the spin state measurement. Both schemes are based on giant circular birefringence and giant Faraday rotation induced by a single electron spin in a microcavity. Our schemes are deterministic and can generate an arbitrary amount of multi-photon entanglement. Following similar procedures, a scheme for a photon-spin quantum interface is proposed.

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Entanglement lies at the heart of quantum mechanics and is a useful resource in quantum information sciences, especially for quantum communications, quantum computation, quantum metrology and quantum networks [1, 2]. Entanglement has been demonstrated in various quantum systems, among which photons are well investigated as an ideal candidate to transmit quantum information and even for quantum information processing [3]. Starting from entangled photon pairs, multi-photon entanglement can be built, and to our knowledge the current record is six-photon entanglement, which is limited by the brightness and photon indistinguishability of entangled photon-pair sources [4]. There exist three main ways to generate polarization entangled photon pairs. One way is via spontaneous parametric processes in nonlinear crystals or fibres where non-deterministic photon pairs are created [5, 6]. The second way is via radiative quantum cascades in a single atom [7] or biexciton in a semiconductor quantum dot (QD) [8, 9], by which deterministic photon pairs are generated. A third way is via single photon mixing at a non-polarizing beam splitter followed by coincidence measurement [10, 11]. However, all these methods demand quantum interference through photon or path indistinguishability, and only one entanglement bit is obtained. An arbitrary amount of entanglement is highly desirable for quantum communications, quantum computation and quantum networks [12].

In our previous work, we proposed a novel optical non-destructive method - giant Faraday rotation - to detect a single electron spin confined in a QD, which can be used to generate remote spin entanglement [13]. In this Letter, we present two novel schemes to generate photon polarization entanglement via single electron spins confined in charged QDs inside microcavities. One scheme is via entangled remote electron spins followed by negatively-charged exciton emissions, and another scheme is via a single electron spin followed by spin state measurement. Both schemes are based on giant circular birefringence and giant Faraday rotation induced by a single electron

spin. Our schemes are deterministic and can be used to generate an arbitrary amount of multi-photon entanglement, but it is not limited by the brightness and photon indistinguishability of entangled photon-pair sources. Besides the spin-photon, spin-spin and photon-photon entanglement, a scheme for a photon-spin quantum interface is proposed. These enable us to make all building blocks for quantum computation and quantum networks based on photons and spins.

The optical properties of singly charged QDs are dominated by the optical transitions of the negatively-charged exciton ( $X^-$ ) which consists of two electrons bound to one hole [14]. Due to the Pauli's exclusion principle,  $X^-$  shows spin-dependent optical transitions [15]: the left-handed circularly polarized photon (marked by  $|L\rangle$  or L-photon) only couples the electron in the spin state  $|\uparrow\rangle$  to  $X^-$  in the spin state  $|\uparrow\downarrow\uparrow\rangle$  with the two electron spins antiparallel; the right-handed circularly polarized photon (marked by  $|R\rangle$  or R-photon) only couples the electron in the spin state  $|\downarrow\rangle$  to  $X^-$  in the spin state  $|\uparrow\downarrow\downarrow\rangle$ . Here  $|\uparrow\rangle$  and  $|\downarrow\rangle$  represent electron spin states  $|\pm\frac{1}{2}\rangle$ ,  $|\uparrow\rangle$  and  $|\downarrow\rangle$  represent heavy-hole spin states  $|\pm\frac{3}{2}\rangle$ .

Now we consider a charged InGaAs/GaAs QD inside a micropillar microcavity with circular cross section (see Fig. 1). The two GaAs/Al(Ga)As distributed Bragg reflectors (DBR) and the transverse index guiding provide the three-dimensional confinement of light. The microcavity is expected to vary or enhance the optical properties of QDs via cavity quantum electrodynamics (cavity-QED). For simplicity, a single-sided cavity is considered here: the bottom DBR is highly reflective while the top DBR is partially reflective in order to couple the light into and out of the cavity. The QD is located at the antinodes of the cavity field to achieve optimized light-matter coupling. Cavity-QED is governed by three parameters:  $g$ ,  $\kappa$  and  $\gamma$ , where  $g$  is the  $X^-$ -cavity coupling strength,  $\kappa$  is twice the cavity field decay rate, and  $\gamma$  is twice the QD dipole decay rate. By solving the Heisenberg equations for the cavity field operator and QD dipole operator in

the approximation of weak excitation, we can calculate the reflection coefficient  $r(\omega) \equiv |r(\omega)|e^{i\varphi(\omega)}$ , where  $|r(\omega)|$  is the reflectance and  $\varphi(\omega)$  is the phase shift. Details can be found in Ref. [13].

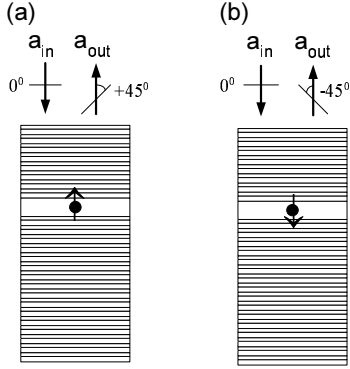


FIG. 1: Schematic diagram of a quantum non-demolition measurement of a single electron spin based on giant optical Faraday rotation. The charged QD is located at the center of a micropillar microcavity and strongly couples to the cavity. In this paper, the Faraday rotation angle is tuned to be  $\pm 45^\circ$  (corresponding to a phase shift  $\Delta\varphi = \pi/2$ ) by setting  $\omega - \omega_c = \kappa/2$ .

For an empty cavity (cold cavity) without a QD inside the cavity or with a QD inside but decoupling to the cavity mode, we obtain  $|r_0(\omega)| = 1$  and  $\varphi_0(\omega) = \pm\pi + 2\arctan 2(\omega - \omega_c)/\kappa$  where “+” stands for the case of  $\omega \leq \omega_c$  and “-” for  $\omega \geq \omega_c$ ,  $\omega_c$  is the cavity mode frequency.  $\varphi_0(\omega)$  can be tuned between  $-\pi$  and  $\pi$  by varying the frequency detuning  $\omega - \omega_c$ . In the strong coupling region with  $g \gg (\kappa, \gamma)$  (we call it hot cavity hereafter), we can get  $|r(\omega)| \simeq 1$  and  $\varphi_h(\omega) \simeq 0$  for small frequency detuning  $|\omega - \omega_c| \ll g$ . If the single excess electron lies in the spin state  $|\uparrow\rangle$  [16], the L-photon feels a hot cavity and gets a phase shift of  $\varphi_h(\omega)$  after reflection, whereas the R-photon feels the cold cavity and gets a phase shift of  $\varphi_0(\omega)$ ; Conversely, if the electron lies in the spin state  $|\downarrow\rangle$ , the R-photon feels a hot cavity and get a phase shift of  $\varphi_h(\omega)$  after reflection, whereas the L-photon feels the cold cavity and gets a phase shift of  $\varphi_0(\omega)$ . We call this phenomenon giant circular birefringence (GCB), which results in giant Faraday rotation (GFR) of linearly polarized light [17]. Both GCB and GFR are induced by a single electron spin in a microcavity due to cavity-QED and the optical spin selection rule of  $X^-$  transitions. GFR provides a quantum non-demolition measurement of a single electron spin (see Fig. 1), whereas GCB can be used to make a phase gate with a phase shift operator defined as

$$\hat{U}(\Delta\varphi) = e^{i\Delta\varphi(|L\rangle\langle L|\otimes|\uparrow\rangle\langle\uparrow| + |R\rangle\langle R|\otimes|\downarrow\rangle\langle\downarrow|)}, \quad (1)$$

where  $\Delta\varphi = \varphi_h(\omega) - \varphi_0(\omega) \simeq -\varphi_0(\omega)$  for small frequency detuning  $|\omega - \omega_c| \ll g$ . In the following, we show that

this phase gate can be used to generate photon-photon entanglement and build a photon-spin quantum interface. Unless otherwise specified, we set  $\Delta\varphi = \frac{\pi}{2}$  by adjusting  $\omega - \omega_c = \kappa/2$  in this paper [18].

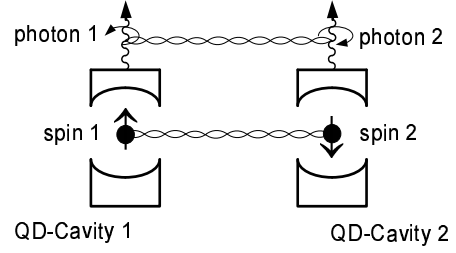


FIG. 2: A proposed scheme to create polarization entanglement between photons via entangled electron spins confined in QDs.

**A. Photon entanglement via entangled remote electron spins.** As discussed in our previous work [13], the spin states  $|\psi_1^s\rangle = \alpha_1|\uparrow\rangle_1 + \beta_1|\downarrow\rangle_1$  and  $|\psi_2^s\rangle = \alpha_2|\uparrow\rangle_2 + \beta_2|\downarrow\rangle_2$  of two electrons confined in two QDs inside two micro-cavities become entangled after interacting with a photon in a linear polarization state, eg.  $|H\rangle \equiv (|R\rangle + |L\rangle)/\sqrt{2}$ . The spin-photon interaction is described by the phase shift operator defined in Eq. (1) with  $\Delta\varphi = \pi/2$ . If detecting the photon in the  $|V\rangle \equiv (|R\rangle - |L\rangle)/\sqrt{2}$  state, the two electron spins are in an entangled state  $|\Phi_{12}^s\rangle = \alpha_1\alpha_2|\uparrow\rangle_1|\uparrow\rangle_2 - \beta_1\beta_2|\downarrow\rangle_1|\downarrow\rangle_2$ . On detecting the photon in the  $|H\rangle$  state, the two electron spins are in another entangled state  $|\Psi_{12}^s\rangle = \alpha_1\beta_2|\uparrow\rangle_1|\downarrow\rangle_2 + \alpha_2\beta_1|\downarrow\rangle_1|\uparrow\rangle_2$ . Here  $H$  and  $V$  denote horizontal and vertical polarization of the photon.

Once entangled spins are prepared, either optical or electrical pumping can be applied to excite  $X^-$  in QDs. Due to the optical spin selection rule of  $X^-$  transitions,  $X^-$  emissions yields two entangled photons in the state

$$|\Phi_{12}^{ph}\rangle = \alpha_1\alpha_2|L\rangle_1|L\rangle_2 - \beta_1\beta_2|R\rangle_1|R\rangle_2, \quad (2)$$

or

$$|\Psi_{12}^{ph}\rangle = \alpha_1\beta_2|L\rangle_1|R\rangle_2 + \alpha_2\beta_1|R\rangle_1|L\rangle_2, \quad (3)$$

depending on the entangled spin states (see Fig. 2).

In this scheme, spin entanglement is directly transferred to photon entanglement via  $X^-$  emissions. Multi-photon entanglement can be built either from multi-spin entanglement [13] or from entangled photon pairs. This is a novel deterministic way to generate polarization entangled photons from single QDs, different from the biexciton cascades [8, 9]. This scheme creates an arbitrary amount of entanglement rather than one entanglement bit, and relies on the long electron spin coherence so that the spin entanglement can persist during the  $X^-$  emission process. This is the case for self-assembled InGaAs/GaAs QDs: the electron spin coherence time ( $\sim \mu\text{s}$ ) limited by the electron spin relaxation

time ( $\sim$ ms) [19], is much longer than the  $X^-$  emission lifetime ( $< 1$  ns) for QDs in the cavity. Therefore, quantum coherence is still essential in the entanglement generation.

As the QD-cavity systems work in the strong coupling regime, the photon emitting is highly efficient [20] and the time jitter between photons is determined by the cavity photon lifetime which is quite short.

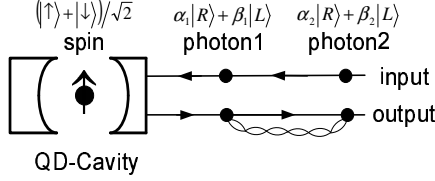


FIG. 3: A proposed scheme to create polarization entanglement between photons via a single electron spin confined in a QD.

**B. Photon entanglement via a single electron spin.** In Fig. 3, photon 1 in the state  $|\psi^{ph}\rangle_1 = \alpha_1|R\rangle_1 + \beta_1|L\rangle_1$  and photon 2 in the state  $|\psi^{ph}\rangle_2 = \alpha_2|R\rangle_2 + \beta_2|L\rangle_2$  are input into the cavity in sequence [21]. Both photons have the same frequency. The electron spin in the QD is prepared in a superposition state  $|\psi^s\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$  and the phase shift operator for the QD-cavity system is described by Eq. (1) with  $\Delta\varphi = \pi/2$ . After reflection, the photon states become entangled with the spin state, and the corresponding state transformation is

$$\begin{aligned} & (\alpha_1|R\rangle_1 + \beta_1|L\rangle_1) \otimes (\alpha_2|R\rangle_2 + \beta_2|L\rangle_2) \otimes (|\uparrow\rangle + |\downarrow\rangle) \\ & \rightarrow (|\uparrow\rangle - |\downarrow\rangle) [\alpha_1\alpha_2|R\rangle_1|R\rangle_2 - \beta_1\beta_2|L\rangle_1|L\rangle_2] \\ & + i(|\uparrow\rangle + |\downarrow\rangle) [\alpha_1\beta_2|R\rangle_1|L\rangle_2 + \alpha_2\beta_1|L\rangle_1|R\rangle_2]. \end{aligned} \quad (4)$$

By applying a Hadamard gate on the electron spin (eg. using a  $\pi/2$  magnetic pulse), the two spin superposition states can be rotated to the states  $|\uparrow\rangle$  and  $|\downarrow\rangle$ . Now the electron spin states can be measured by the GFR-based quantum non-demolition method shown in Fig. 1. Photon 3 in the state  $(|R\rangle_3 + |L\rangle_3)/\sqrt{2}$  is input into the cavity (photon 3 has the same frequency as photon 1 and 2), after reflection, the total state for the three photons and one spin becomes

$$\begin{aligned} & (|R\rangle_3 + i|L\rangle_3)|\uparrow\rangle [\alpha_1\alpha_2|R\rangle_1|R\rangle_2 - \beta_1\beta_2|L\rangle_1|L\rangle_2] \\ & - (|R\rangle_3 - i|L\rangle_3)|\downarrow\rangle [\alpha_1\beta_2|R\rangle_1|L\rangle_2 + \alpha_2\beta_1|L\rangle_1|R\rangle_2]. \end{aligned} \quad (5)$$

The output state of photon 3 can be measured in orthogonal linear polarizations. If the photon 3 is detected in the  $|R\rangle_3 + i|L\rangle_3$  state ( $45^\circ$  linear), so the electron spin is definitely in the state  $|\uparrow\rangle$  and we project Eq. (5) onto an entangled photon pair state

$$|\Phi_{12}^{ph}\rangle = \alpha_1\alpha_2|R\rangle_1|R\rangle_2 - \beta_1\beta_2|L\rangle_1|L\rangle_2. \quad (6)$$

On detecting the photon 3 in the  $|R\rangle_3 - i|L\rangle_3$  state ( $-45^\circ$  linear), so the spin is definitely in the state  $|\downarrow\rangle$  and we project Eq. (5) onto another entangled photon pair state

$$|\Psi_{12}^{ph}\rangle = \alpha_1\beta_2|R\rangle_1|L\rangle_2 + \alpha_2\beta_1|L\rangle_1|R\rangle_2. \quad (7)$$

Obviously, an arbitrary amount of polarization entanglement between photons is obtained. Multi-photon entanglement can be built either by interacting multiple independent photons with a single electron spin, or from entangled photon pairs. Similar to scheme A, this scheme is deterministic and relies on the long electron spin coherence, i.e., the time difference between photons should be shorter than the electron spin coherence time ( $\sim \mu$ s) in QDs.

Scheme A and B are both based on giant circular birefringence and giant Faraday rotation. In the following, we show that a photon-spin quantum interface can be built by applying the same phase gate  $\hat{U}(\Delta\varphi)$  and single-spin detection method.

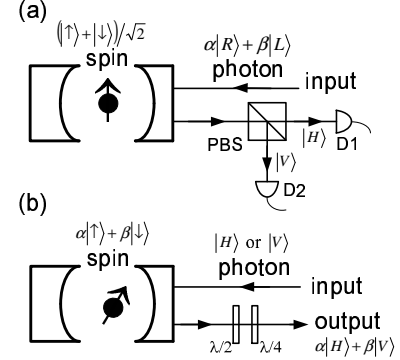


FIG. 4: A proposed scheme to transfer quantum state (a) from a photon to an electron spin, and (b) from an electron spin to a photon. PBS - polarizing beam splitter; D1, D2 - photon detectors;  $\lambda/2$ ,  $\lambda/4$  - half-, quarter-wave plates.

### C. Quantum state transfer from photon to spin.

In Fig. 4(a), a photon in an unknown state  $|\psi^{ph}\rangle = \alpha|R\rangle + \beta|L\rangle$  is input to a QD-cavity system with the electron spin prepared in the state  $|\psi^s\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ , and a phase shift operator  $\hat{U}(\Delta\varphi = \pi/2)$ . After reflection, the photon state and spin state become entangled, i.e.,

$$\begin{aligned} & (\alpha|R\rangle + \beta|L\rangle) \otimes (|\uparrow\rangle + |\downarrow\rangle) \rightarrow \\ & \alpha|R\rangle|\uparrow\rangle + i\alpha|R\rangle|\downarrow\rangle + i\beta|L\rangle|\uparrow\rangle + \beta|L\rangle|\downarrow\rangle. \end{aligned} \quad (8)$$

By applying a Hadamard gate on the photon state (eg. using a polarizing beam splitter), we obtain a spin state  $|\Phi_1^s\rangle = \alpha(|\uparrow\rangle + i|\downarrow\rangle) + i\beta(|\uparrow\rangle - i|\downarrow\rangle)$  if detecting a photon in the  $|H\rangle$  state, or a spin state  $|\Psi_1^s\rangle = \alpha(|\uparrow\rangle + i|\downarrow\rangle) - i\beta(|\uparrow\rangle - i|\downarrow\rangle)$  if detecting a photon in the  $|V\rangle$  state. Next, by applying a Hadamard gate on the electron spin (eg. using a  $\pi/2$  magnetic pulse), the electron spin state is converted to  $|\Phi_1^s\rangle = \alpha|\uparrow\rangle + i\beta|\downarrow\rangle$  and  $|\Psi_1^s\rangle = \alpha|\uparrow$

$\rangle - i\beta|\downarrow\rangle$ . By applying a unitary operation on the electron spin states, we get  $|\psi^s\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$ . Therefore, a unknown photon state is transferred to the electron spin state.

#### D. Quantum state transfer from spin to photon.

In Fig. 4(b), a photon in the polarization state  $|\psi^{ph}\rangle = (|R\rangle + |L\rangle)/\sqrt{2}$  is input to a QD-cavity system with the electron spin in a unknown state  $|\psi^s\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$ , and a phase shift operator  $\hat{U}(\Delta\varphi = \pi/2)$ . After reflection, the photon state and spin state become entangled, i.e.,

$$(|R\rangle + |L\rangle) \otimes (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \rightarrow \alpha(|R\rangle + i|L\rangle)|\uparrow\rangle + \beta(i|R\rangle + |L\rangle)|\downarrow\rangle. \quad (9)$$

After applying a Hadamard gate on the electron spin (eg. using a  $\pi/2$  magnetic pulse), a third photon in the state  $(|R\rangle + |L\rangle)/\sqrt{2}$  is input to measure the electron spin state by using the GFR-based quantum non-demolition measurement (see Fig. 1). If detecting the electron spin in the  $|\uparrow\rangle$  state, the photon is then projected in the state  $|\Phi_1^{ph}\rangle = \alpha|+45^\circ\rangle + i\beta|-45^\circ\rangle$ ; If detecting the electron spin in the  $|\downarrow\rangle$  state, the photon is then projected in the state  $|\Psi_1^{ph}\rangle = \alpha|+45^\circ\rangle - i\beta|-45^\circ\rangle$ . Here  $|\pm 45^\circ\rangle \equiv (|R\rangle \pm i|L\rangle)/\sqrt{2}$ . By applying a unitary operation (eg. using a  $\lambda/2$  and a  $\lambda/4$  wave plate) on the photon state, we get  $|\psi^{ph}\rangle = \alpha|H\rangle + \beta|V\rangle$  [21]. So a unknown spin state is transferred to the photon state.

Different from the original teleportation protocol which involves three qubits [22], our state-transfer scheme needs only two qubits thanks to an arbitrary amount of spin-photon entanglement achieved. The state-transfer fidelity is affected by the imperfect reflectance of the hot cavity, i.e.,  $|r(\omega)| \rightarrow 1$  when  $|\omega - \omega_c| \ll g$ , and the asymmetric shape of QDs in practice.

In conclusion, we have presented two novel schemes to generate an arbitrary amount of photon polarization entanglement via single electron spins confined in charged QDs inside microcavities based on the giant circular birefringence and giant Faraday rotation induced by a single electron spin. Both schemes are deterministic and rely on the electron spin coherence, indicating that quantum coherence remains essential to the entanglement generation. Our schemes can be used to generate multi-photon entanglement, but it is not limited by the brightness and photon indistinguishability of entangled photon-pair sources. A scheme of a quantum interface between photon and spin is also proposed. With the spin-photon, spin-spin and photon-photon entanglement as well as the photon-spin quantum interface, we can make all building blocks, such as quantum memories, quantum repeaters and various quantum logic gates for quantum communi-

cations, quantum computation and quantum networks. We believe this work opens a new avenue in quantum information sciences.

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